

Chapter 2: Basic physics concepts and simple harmonic motion (SHM)

In this chapter, I will introduce some basic concepts in *mechanics*. These include speed, velocity, and acceleration, forces, Newton's three laws of motion, and energy. The treatment here will not be as thorough as I could make it; you should consult an introductory physics text or [online resource](#) for more information.

Why bother introducing mechanics? Well, a traveling sound transmits energy, so it would help to understand what energy is like in a few contexts. Also, patterns that appear in the motion of physical systems such as pendulums and mass/spring systems appear frequently in our study of sound waves.

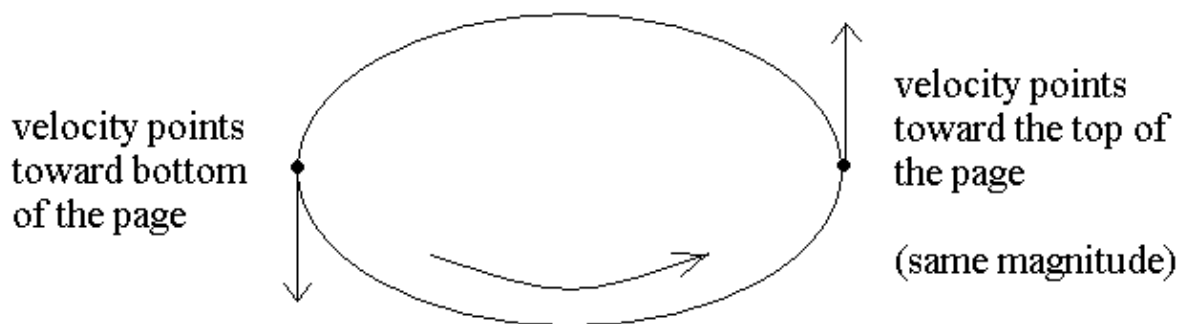
Introductory mechanics concepts

Speed and velocity

If you've heard the term *velocity* before, you may wonder how it differs from the more commonly used term *speed*. Though the ideas are closely related, since finding either involves looking at some change in position over a time interval, a velocity and a speed are different types of quantities. Velocity is an example of a *vector* quantity. Informally, you can think of vectors as arrows with two features: a size (called the *magnitude*) and a direction. Speed represents the magnitude of a velocity vector; that is the connection between the two terms.

When we get into specific definitions, things can get complicated. The definition of *average velocity* is the *displacement* one travels divided by the time it takes. *Displacement* is defined as the difference between one's final and initial position. It is a vector, and its magnitude is the distance "as the crow flies" from the starting point to the ending point. The definition of *average speed* is the *distance* one travels divided by the time it takes.

The difference between average velocity and average speed: To illustrate the difference, think about a runner training for the Olympics who can run once around a 400 m track in 50 seconds. That won't win you any medals, but it's very fast! If you consider one complete lap, the distance the runner travels is 400 m, so his average speed is $(400 \text{ m}) / (50 \text{ s}) = 8 \text{ m/s}$. However, if you consider his starting and ending points ONLY, you'll see they're the same position, so his total *displacement* for the lap is zero. Therefore, his average velocity is zero! The way this can happen is that at one point along the track, his velocity at *that instant* is 8 m/s to the east, say. At another point along the track, his velocity at that instant is 8 m/s to the west. In averaging his velocity all along the trip, these opposing velocity vectors cancel out.



A picture showing how motion in a circular path can lead to zero average velocity. The two vectors shown here cancel out when you average them together.

Most of the time in this class, we consider motion in just one direction at a time. If motion only happens in one direction, the magnitude of the average velocity and the average speed will be the same number.

Units: Typical units for speed in the U.S. are miles per hour (mph or mi/hr) or feet per second (ft/s).

Note that units for both involve a distance unit on top and a time unit on the bottom. In the SI system, we follow the same pattern to see that both speed and the magnitude of velocity vectors is written in meters per second, or m/s. Smaller speeds can be written in units like cm/s.

Acceleration

When I teach physics to introductory students, the example of acceleration they almost always bring up is that of cars. If you watch a car commercial (particularly for fast or sporty cars), you'll hear them brag about how quickly the car can go from zero to 60 mph. The claim will be something like "Car X goes from 0-60 in 5 seconds." That information is all you need to calculate one type of *acceleration*.

Acceleration is defined as the change in velocity over the amount of time it takes for the change. Many physics books write it like this:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Acceleration, like velocity, is a vector quantity. The triangle symbol is the Greek letter delta, and it represents a change in a quantity; the numerator here would be written as (final velocity – initial velocity) and the denominator can be written as (final clock reading – initial clock reading).

More than one kind of acceleration: Most people picture acceleration as something you do when you speed up, and that's certainly right. However, slowing down (commonly called *deceleration*) also counts as an acceleration in the physics sense – sometimes that's given a negative number, but it depends on your coordinate system. We can go farther, though. Recall ANY change in velocity is by definition non-zero acceleration. Therefore, if you change anything about your velocity vector, you're accelerating, and that includes the direction. So let's say you maintain constant speed in a car, but you turn the steering

wheel. That's acceleration too! Keep that in mind when your friends tell you that your car only has one accelerator.

Units: The units of acceleration are the same as the unit of velocity divided by the unit of time, which is sometimes written m/s/s, but is more often written m/s², so it's clear that both "factors" of time are in the denominator.

For the cases where something moves in one direction and speeds up or slows down, the magnitude of the acceleration vector tells you how fast the speed is changing. Suppose a car starts from rest (zero speed) and the driver hits the gas. If the car's acceleration is, say, 2 m/s², that means it will be traveling 2 m/s after one second, 4 m/s after two seconds, 6 m/s after three seconds, and so on.

Newton's Laws of Motion

Now that you have an idea about what acceleration is, let's connect that to Newton's Laws of Motion, named after Sir Isaac Newton (1642-1727). Many of you have heard his laws of motion as early as elementary school. Versions of two of his laws often read like this:

Newton's 1st Law (simplified): An object at rest stays at rest and an object in motion stays in motion.

Newton's 3rd Law (simplified): For every action, there is an equal and opposite reaction.

Most physics texts have multiple chapters on Newton's Laws and their use in force problems. Here, I just want to introduce a slightly more specific form of the three laws and use them to discuss the idea of weight.

Newton's 1st Law: An object will remain in its current state of motion (either steady velocity or at rest) unless a nonzero **net** force acts on it. The *net force* mentioned here is the sum of all forces acting on an object – and remember, forces are vectors, so they can cancel if they point in opposite directions.

Newton's 2nd Law: The amount a mass will accelerate is equal to the net force it feels divided by its mass. Mathematically, this is written:

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

I write this in terms of acceleration and not net force since I want to illustrate here that an acceleration is an effect on an object due to a nonzero net force. Note that Newton's 1st Law is a special case of Newton's 2nd Law where net force and acceleration are zero.

Newton's 3rd Law: Forces occur in pairs. If object A exerts a force on object B, B exerts a force on A that's equal in magnitude and opposite in direction.

This is often a counterintuitive idea. One implication of Newton's 3rd law says that when a large truck runs into a parked car and smashes it, the force the truck exerts on the car and the force the car exerts

on the truck are equal. How can this be true? Clearly, the car will get smashed, and you'd rather be sitting in the truck if you had to be in one vehicle or the other – so what's going on here?

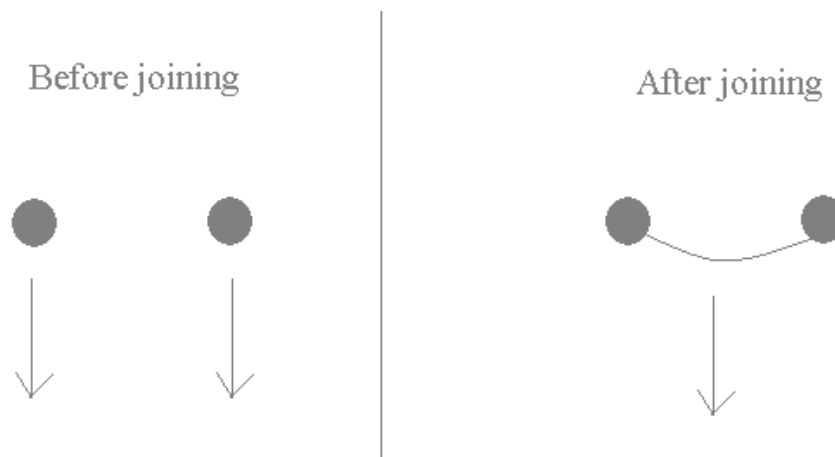
If you consider Newton's 2nd Law, this becomes plausible. Looking at the accelerations of each vehicle, you can see if they feel the same force, the truck's larger mass will result in it experiencing less acceleration than the car. This accounts for the difference in the "reaction" a viewer will notice in the collision.

Gravity and weight

In the last chapter, I mentioned that we must be careful to distinguish mass and weight. Consider a person who weighs 150 pounds on Earth. On the moon, she weighs only 25 pounds, and it is because the moon exerts a smaller gravitational force on her than Earth does. An object's *weight* is defined as the gravitational force acting on it. Now, my hypothetical person will have the same amount of matter present whether on Earth or the moon, so the mass will be about 68 kg on either surface. On earth, a mass of 1 kg has a weight of about 2.2 pounds.

Note on force units: Newton's 2nd Law implies that the net force something feels equals its mass times its acceleration. Therefore, the most basic SI unit of force multiplies those two units. 1 kg m/s² is given the name 1 Newton.

Galileo and falling: What happens if you drop anything from a height? You should notice that in addition to simply falling, it will *speed up* as well. A common story from the history of physics fits here. As the story goes, Galileo took two vastly different weights to the top of the Leaning Tower of Pisa in Italy and dropped them – thereby discovering that they hit the ground at the same time. This is likely folklore, though he did conduct a thought experiment in this vein. Imagine dropping two balls as he may have. They'll fall together. His thought experiment was this: suppose in mid-fall that you link the two together with a light string or strand of spider web. Is it then a double-mass object? Would it immediately start falling twice as quickly? Clearly not! Of course, being an experimentalist, Galileo DID test this thought by rolling different masses down inclined planes and timing their movements with a water clock.



Acceleration due to gravity

There are many experimental ways to measure the acceleration due to gravity. You could drop something from a height, time the fall, and use *kinematic equations* (covered in basic physics classes, but not here) to figure out the acceleration. There are also ways of finding the acceleration doing an incline plane experiment like Galileo did or by timing a pendulum. At any rate, near Earth's surface, the *acceleration due to gravity* has a value of about 9.8 m/s^2 . Anything will accelerate downward at this rate if under the force of gravity only. You can even see that a [feather will fall identically](#) to a heavier object in a vacuum. Though the acceleration due to gravity is different on the Moon, [astronauts have done this](#) demo also. Normally, a feather or a piece of tissue will not accelerate this way near Earth, but this is due to air resistance.

If you consider something in free fall (that is, with only gravity acting on it), you can use Newton's 2nd Law to find how much force must be acting. The *acceleration due to gravity* is given the symbol "g." If you note that $g = \frac{F_{net}}{m}$ and that the net force and gravity are the same in this case, you can multiply both sides by m to get

$$F_g = mg.$$

This is the connection between mass and weight. If mass is in kilograms and acceleration due to gravity is in m/s^2 , the resulting force will be in Newtons. Using the value given above, I like to remember the verse: "if you want to convert mass to weight / multiply mass by 9.8." Of course, you have to make sure everything is in the right units.

Energy

Types of energy

Energy is often defined as the ability to do work, but there are a lot of contexts this work can be done in (and there's another problem – physics has a [strict definition](#) of work; see #13). Roughly speaking, if you do work on an object, you give it energy. Work and energy are the same type of quantity with the same units. I am willing to bet most readers have come across different types of energy in previous school discussions and in everyday life.

How many types can you think of? Try listing a few.

Go ahead. I'll wait.

Some popular types of energy that come up in lists like this include heat energy, sound energy, electrical energy, mechanical energy, or chemical energy. You may even list one of the major types that all these examples fall into: *potential energy* or *kinetic energy*. Potential energy is energy an object has stored in it somehow, and often the energy storage is due somehow to its position. Kinetic energy is the energy something has due to its motion.

Units: The physics definition of work states that when you exert a force over a distance, work is done. The work done by a force is the product of the distance an object travels with the force exerted *in that direction*. Therefore, the unit of work and energy is a Newton meter ($1 \text{ N}\cdot\text{m}$), which is given the name Joule (J). Another popular energy unit is the calorie (cal), which is defined as the amount of heat energy needed to raise the temperature of one gram of water by 1°C . There are roughly 4.18 Joules in a calorie.

Conservation of energy

One of the reasons that it's worth it to get into the mathematics of energy is that *energy is conserved*, that is, it cannot be created or destroyed. There are many ways it can change form in some physical process, but that's OK as long as we can keep track of it. As a student in a physics class, you should be aware that some of the more interesting and insightful treatments of introductory physics topics were written by Richard Feynman in 1964 while teaching at Caltech. [Here](#) is his discussion of energy conservation – you can read the [rest of his lectures](#) online for free as well if you have the time and inclination.

It turns out that we can quantify a few common types of energy and use that refinement to figure out what goes on in a simple energy transfer situation. I will discuss those energy types in the next section and give examples of how we can use the conservation of energy to solve certain problems.

Types of potential and kinetic energy we will deal with in this chapter

Kinetic energy is probably the simplest form to start with, since anything that has mass and moves has it. A study of kinematics is required to derive the formula, so I will simply quote it here. For masses traveling “non-relativistically,” that is, much slower than the speed of light, the kinetic energy they have is given by

$$KE = \frac{1}{2}mv^2$$

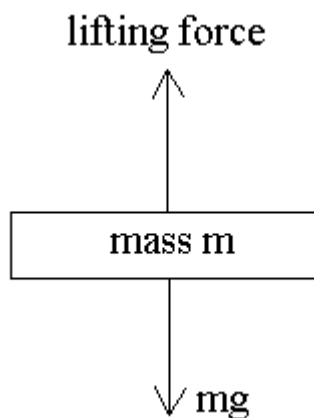
where KE stands for kinetic energy, m is the mass of the object, and v is the magnitude of the object’s velocity. Effectively, v stands for speed.

Features: If SI measurements are used, where mass is in kilograms and velocity is in meters per second, the combined unit of energy is the kg (m/s)^2 . This is cumbersome to write out, so it is given a name – the *Joule* (or J for short). This unit is named after James Prescott Joule, an English scientist who studied the relationship between work and heat.

It is also worth noting that since for ordinary matter (as opposed to antimatter), mass is always a positive quantity. Since the square of any velocity is also positive, the kinetic energy of any object is necessarily greater than or equal to zero.

Gravitational potential energy is the energy present due to the relative position between masses exerting a force of gravity on each other. The easiest way to visualize this is by picturing the drop of a rock or ball – the higher up you lift it, the faster it will be going when it hits the ground. It has the capacity to gain this extra speed due to the potential energy it has.

Near Earth, it is not too hard to figure out how much gravitational potential energy something has. Suppose you use a force to lift a mass m through a height h . To lift it at constant speed (therefore keeping the kinetic energy constant), you must use a force equal to mg (its weight). Using the definition of work as the product of force and distance (acting in the same direction), one finds you do mgh worth of work.



Force diagram showing an object as it is being lifted. It may seem weird the forces are equal, but any motion with zero acceleration requires balanced forces.

In summary, when near Earth in a space where the gravitational acceleration is approximately constant, the gravitational potential energy possessed by a mass is given by the equation

$$PE_g = mgh$$

where m is the mass, g the acceleration due to gravity (9.8 m/s^2 near Earth), and h is the height above some arbitrary reference level. In discussing potential energy, only *differences* between heights matter, so you can technically call the ground level the “zero height” point if you want, or you can call the floor of whatever room you’re in the “zero height” location – just be consistent within a problem!

Spring potential energy is another type of stored energy. In this course, we will be doing a lab with masses and springs, and you should know (especially if you’ve played pinball) that a spring stores energy if you stretch it or compress it to a length other than its “natural” length. This is because it takes force to stretch or compress a spring - a result known as *Hooke’s Law*.

$$F_{\text{spring}} = -kx$$

In this equation, the spring force is proportional to x , the amount of stretch or compression from the natural length. The constant k in the equation is called the *spring constant*, and it is larger for stiffer springs. The unit of spring constants in the SI system is the Newton per meter (N/m). The negative sign simply means that if you stretch a spring one way, the force it exerts points in the other direction.

The potential energy a spring stores can be determined as well – this formula is

$$PE_{\text{spring}} = \frac{1}{2}kx^2$$

You should check that both PE formulas give units equal to a Joule when the standard kilogram, meter, and second units are inserted. All forms of energy had better be in the same units before we talk about converting between them.

Simple energy conservation



All PE
No KE
 $v = 0$

All KE
 $v = ?$



The problem in question

A normal piece of paper will not fall to the ground smoothly. However, if you crumple up a piece, it will fall somewhat normally. We might ask the question – if I drop a piece of paper from 1.5 meters high, how fast will it be going just as it's about to hit the ground (provided we neglect air resistance)?

A piece of paper has a mass of about 5 grams. That's equivalent to 0.005 kg. Therefore, at the moment of release, when all the energy is in the form of potential energy, we get an energy of

$$PE = mgh$$

$$PE = (0.005 \text{ kg}) \times \left(9.8 \frac{\text{m}}{\text{s}^2}\right) \times (1.5 \text{ m}) = 0.0735 \text{ J}$$

We make the assumption that air resistance is negligible. Therefore, close to impact, all the potential energy will have been converted to kinetic energy. Since we know the formula for kinetic energy, we can solve for the final velocity!

$$0.0735 \text{ J} = \frac{1}{2} * (0.005 \text{ kg}) * v^2$$

$$\frac{0.0735 \text{ J}}{0.0025 \text{ kg}} = v^2$$

$$\sqrt{\frac{0.0735 \text{ J}}{0.0025 \text{ kg}}} = v$$

$$v = 5.42 \text{ m/s}$$

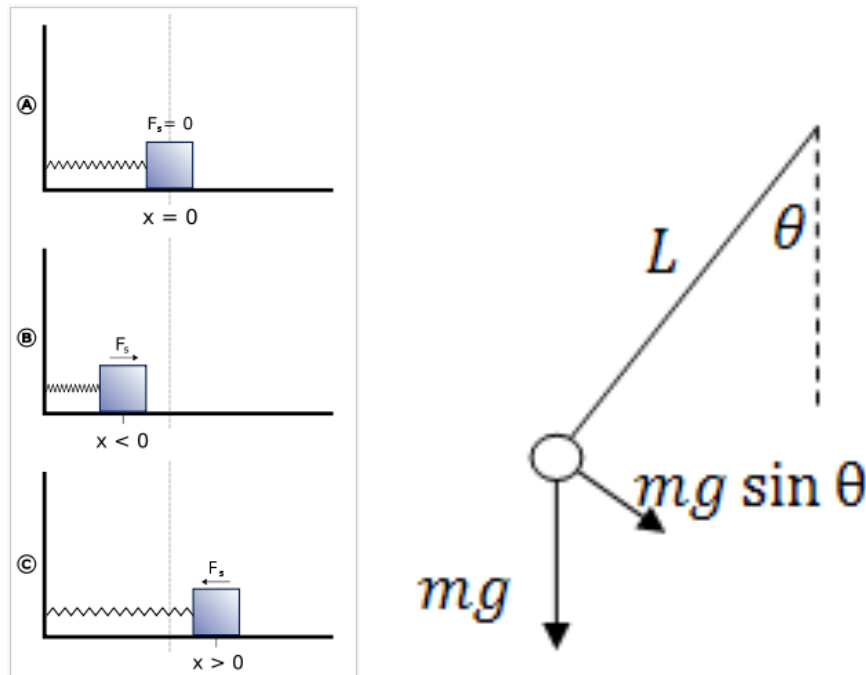
Math trick: Notice that I just set the initial potential energy equal to the final kinetic energy. That's OK because I assumed the floor was a position of zero potential energy, and the paper wasn't moving at the instant of release. If you set the formulas for PE and KE equal to one another, you'll find the masses cancel out! That means that (in the absence of air resistance) all masses fall the same way – which we can see in [this video](#).

Simple Harmonic Motion (SHM)

Now that I've discussed Newton's Laws, forces, and energy somewhat, we can start discussing oscillations and vibrations. Two specific examples are the mass-spring system and the pendulum. These may seem like idealized or possibly irrelevant systems to a student of acoustics, but there are many parallels between patterns seen in these systems and those seen in sound waves.

Definition of simple harmonic motion

Simple harmonic motion is a type of repeating motion that occurs anywhere there is a “restoring” force acting on an object that is proportional to its displacement from equilibrium. Hooke’s Law (described above as $F_{spring} = -kx$) is a perfect example. The simple pendulum is also a good example. Gravity acts on a pendulum bob at all times, but the farther you pull the pendulum back, the less the string tension cancels it out. Gravity acts as a *restoring* force because its job is to get the pendulum back to the central point.

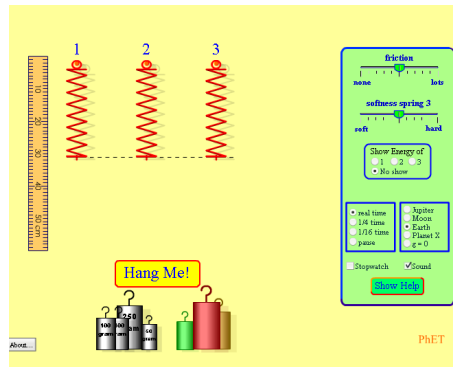


At left, a [mass-spring system](#). The force arrow indicates a restoring force toward equilibrium. At right, a [pendulum](#). The pendulum bob always feels the “ mg ” force of gravity downward, but the tension force (not drawn) partially cancels it, leaving “ $mg \sin \theta$ ” worth of force left over. The larger the angle, the greater the component of non-canceled gravitational force.

The mass-spring system

The oscillating mass spring system is started by pulling the mass to a position where the spring is compressed or stretched from its equilibrium point. The maximum distance the mass gets from equilibrium is called the *amplitude* of the oscillation.

One way to play with systems like this is by using a simulator online like [this one](#) for masses and springs:



Mass and spring simulator

Some activity ideas:

- 1.) Place different masses on the springs. What do you notice about the stretch?
- 2.) Let the masses oscillate up and down. Play with the friction – what happens there?
- 3.) What happens as you move the simulation to a different moon or planet?
- 4.) Which masses go up and down the fastest – or take the least time to make a complete trip?
- 5.) You can make graphs of the energy present in the spring. When does the mass/spring system have the most spring potential energy?
- 6.) When does the mass have the most kinetic energy?
- 7.) Where does the energy go if there is friction?

Lab activity

If you have masses and springs in your class, you can do a lab activity. An interesting thing to investigate is the *period* of an oscillation, or rather, the amount of time it takes for the system to go through one complete cycle. Since period is a time interval, its typical SI unit is the second.

What things could possibly affect the period? Commonly, people say that the spring chosen would make a difference. A stiffer spring would exert larger forces for a given displacement, and that would cause the mass to accelerate and move faster. Similarly, many believe that the choice of mass matters. A larger mass will not experience accelerations that are as large as a smaller mass would. Finally, another testing option is varying the amplitude. If the amplitude of a vibration increases, the mass has to travel through a larger distance.

In the interest of not spoiling your experiments, I will not indicate the final results we get yet. In a few pages, I'll mention the formula for a mass-spring system period and confirm the correct results.

Pendulum system

The good folks at Colorado provide a [simulation for a pendulum system](#) as well. You can also do a pendulum experiment with a real mass and string. Many of the activities I mentioned for the mass and spring above are adaptable to the pendulum applet as well.

Possible variables that COULD affect the period of a pendulum include the amplitude (how far back you pull the pendulum), the mass of the bob, and the length of the string. Again, I will save the result for later in case you want to try the simulation or experiment for yourself.

Other physical systems exhibit behavior similar to SHM. Many are acoustically relevant. For example, the prongs of a tuning fork move back and forth in a similar pattern. To produce tones we can hear, though, a tuning fork has to move back and forth much more quickly than the mass-spring or pendulum systems.

Period and frequency

I've already defined the *period* of a system in simple harmonic motion as the amount of time it takes the system to go through one complete cycle. In acoustics, we are more often interested in the *frequency* of a vibration. Frequency is the number of complete cycles a vibrating system goes through per second. Note that period is akin to "seconds per cycle" and frequency is "cycles per second," so mathematically they are reciprocals of each other. If I write period as T (chosen because it is a *time*) and frequency as f , this can be written:

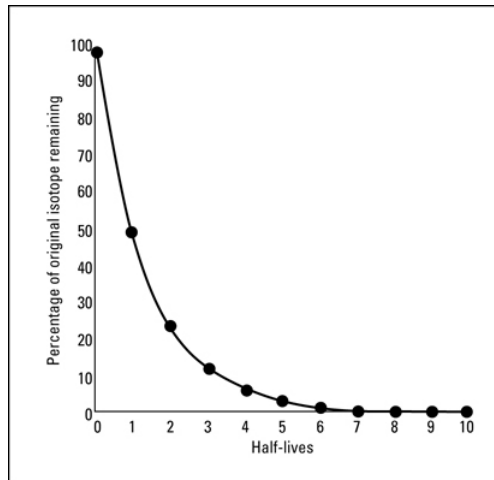
$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}.$$

The unit of frequency is the Hertz (Hz) if period is written in seconds – so Hertz is another way of saying "cycles per second."

Decaying SHM

If you let a pendulum swing for a while, eventually the vibration will die away due to air resistance among other things. A mass-spring system will do the same thing in the "real world." In our new terms, we might say that the amplitude decays away over time.

It is easy to see and understand that, but let us refine. We should know *how* the decay takes place. In cases of drag, the resistive force is proportional to speed – so the larger the amplitude is, the more quickly it will decay. The final result is that amplitudes *exponentially* decay.



A graph of [radioactive exponential decay](#) – the same pattern as SHM amplitude decay!

This is the exact same thing that happens with radioactive elements. The more of an element you have, the more will decay within a given time. To describe how quickly radioactive elements decay, we think of their *half-life*, that is, the amount of time it takes for half the original element to decay away. For example, the half-life of carbon-14 is about 5700 years. If you start with a sample of, let's say 1 mg of carbon-14, then you'll have 0.5 mg 5700 years from now and 0.25 mg 11400 years (two half-lives) from now.

Instead of talking about SHM in terms of half-life, we use the term *decay half-time*. Decay half-time is the amount of time it takes a given oscillating system to drop in amplitude by half, and it usually depends on the surrounding air and system pieces (masses, springs, etc.) you use.

Example: Suppose a mass-spring system has an amplitude of 4 cm. If I tell you the decay half-time is 20 seconds, then every 20 seconds, you'd expect the amplitude to drop by half. In table form, we have:

Time elapsed (s)	Amplitude (cm)
0	4
20	2
40	1
60	0.5
80	0.25
100	0.125
120 (2 min)	0.06125

Decay half-time example: DHT 20 s, starting amplitude 4 cm

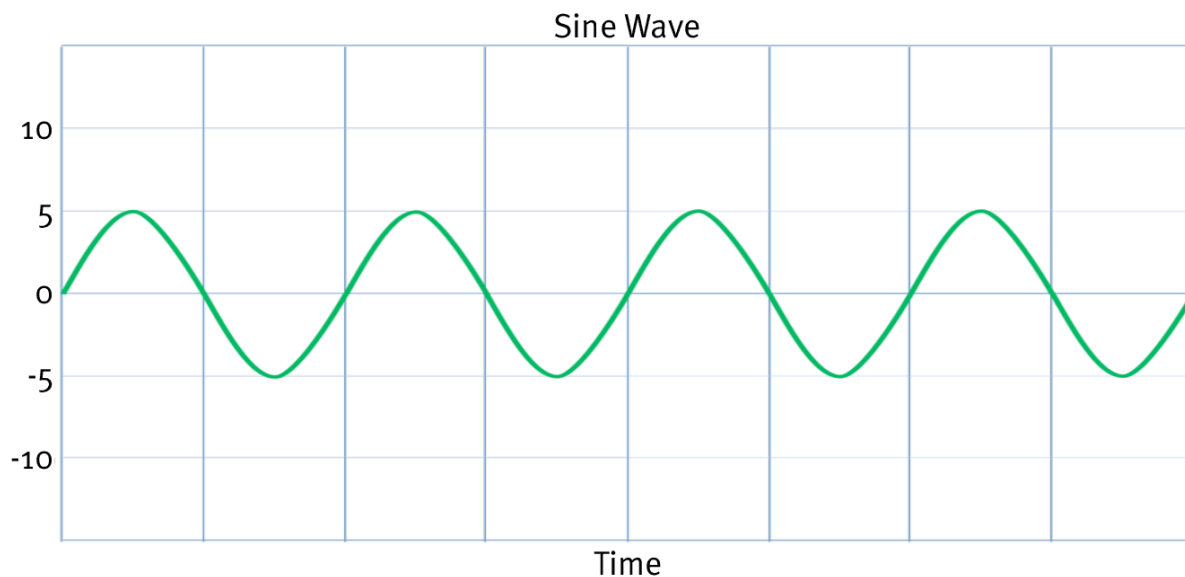
SHM, sine waves, uniform circular motion, and a connection to acoustics

Simple harmonic motion and uniform circular motion are linked. One good way to see that is through [this link](#). This PDF can't show you the animation, but it should work if you click on it. If you have

something moving in a circular path at constant speed, it is possible to map a mass-spring system directly next to it so that their vertical positions are aligned.

Circular motion and the graph for what is called the *sine function* are also linked, as can be seen [here](#). As the arrows trace their way around the circle, you get three different but similarly-patterned sine waves. An even more dramatic video can be found [here](#). In this demonstration, a spray paint can is turned on, and when a paper roll is turned, the paint shows the sine wave pattern.

Sine waves are functions often introduced in trigonometry class, but they are visible throughout nature, and they're very relevant to acoustics. When we start describing waves both on strings and in tubes, the sine wave will be used a lot to draw what is happening because it represents exactly the patterns that form. Sound is a wave in air, and for what are called *pure tones*, the pattern of compressed and rarified air would look exactly like a sine graph if you were to draw it!



A typical sine wave pattern ([Source](#))

It is useful to be able to express wave patterns mathematically. The next chapter contains more details on traveling waves and the “guts” of wave expressions, but for now, know that many sine waves are expressed in this form:

$$f(t) = A \sin (\omega t)$$

where A is the amplitude of the wave and the symbol omega (ω) is connected to the wave's frequency – technically this is called an *angular frequency* and is proportional to f . Chapter 3 introduces concepts like *wavelength*, which also gets its own role in such math. The figure pictured on the last page has an amplitude of 5 in whatever units the y-axis is depicted in, so $A = 5$ in the equation above.

Equations of motion and period of SHM

Time to reveal some spoilers! In the mass-spring system, the mass and spring matter in determining the period. It turns out the amplitude does NOT! It makes sense to figure that greater amplitude means greater total distance to travel. However, if you think about stretching a spring farther, it will exert larger forces and the mass will move FASTER. The greater distance and faster movement exactly cancel each other out.

I'll remind you here that the math supplement chapter of these notes contains review and elaboration on trigonometric functions. It is possible to show that ω for a mass-spring system is given by the square root of k/m . We can say, therefore, that the *equation of motion* giving the position of a mass-spring system is

$$x(t) = A \sin \left(\sqrt{\frac{k}{m}} t + \phi \right)$$

The new symbol ϕ at the end is called a phase and that shifts the function left or right. Since the period of a sine or cosine function $x = \sin Bt$ or $x = \cos Bt$ is $2\pi/B$, the period of a mass-spring system is given by:

$$T_{spring} = 2\pi \sqrt{\frac{m}{k}}$$

A greater mass will result in a larger acceleration, and a greater spring constant (remember, that's k), will result in a smaller period.

If you're curious about the maximum velocity of a mass in a mass-spring system, there are two ways to think about it. One is to use conservation of energy; when you stretch a spring to the maximum amplitude, potential energy is stored in the spring, and when the mass is at equilibrium, this energy has been converted to kinetic energy.

Mathematically, we can find an equation for the velocity as well. This is one of these things that requires calculus to prove, but if $x(t)$ above is my function for the mass's position, the function for the mass's velocity is given by

$$v(t) = A \sqrt{\frac{k}{m}} \cos \left(\sqrt{\frac{k}{m}} t + \phi \right)$$

and $A \sqrt{\frac{k}{m}}$ is the maximum speed of the oscillator in this case since the cosine function outputs values from -1 to 1.

In a pendulum system, the mass does *not* matter (and it shouldn't be a surprise – different masses often fall at the same rate!) Amplitude, as it turns out, does matter, but not much. You get larger speeds if

the amplitude is larger. However, we do find that if the amplitude is small, your exact amplitude doesn't matter much. It also matters what planet you're on (though sadly many of you won't get to do this experiment for yourself). The pendulum period (for small amplitudes) is given by:

$$T_{\text{pend}} = 2\pi \sqrt{\frac{l}{g}}$$

where l is the pendulum length and g the acceleration due to gravity.

Dimensional analysis check

The above formulae can be checked or even nearly derived using dimensional analysis. I'll do this for the mass/spring system, which is the trickier of the two. Recall that the SI unit of force is the Newton, which is equivalent to $1 \text{ kg} \cdot \text{m/s}^2$. That breaks down dimensionally as $[M][L]/[T^2]$. In our discussion of Hooke's Law earlier, we saw that the spring constant k can be written in units of Newtons per meter. That cancels out the dimension of length from the Newton unit, so dimensionally (remember that the brackets indicate we only care about mass, length, and time breakdowns of the enclosed quantities):

$$[k] = [M]/[T^2]$$

Suppose I want to get a quantity with dimensions of time from spring constant somehow. One step might be to divide spring constant by mass. Dimensionally, that gives:

$$\left[\frac{k}{m}\right] = 1/[T^2]$$

That's not quite what we want. Inverting the fraction, we see the dimension of m/k is $[T^2]$, and:

$$\left[\sqrt{\frac{m}{k}}\right] = [T]$$

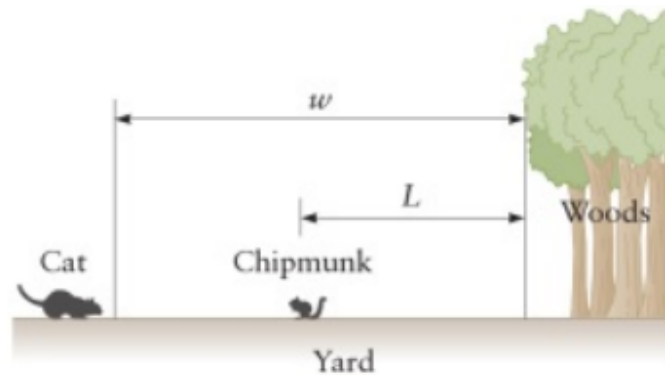
This is very similar to our mass-spring period formula above! Only a factor of 2π is missing, and that is a necessary evil; dimensional analysis cannot derive or tell you anything about dimensionless constants in equations (factors of 2, $\frac{1}{2}$, π , etc.) When my memory fails in a formula like this regarding what variable is in the numerator or denominator, I frequently do a quick dimension check to verify that I have a reasonable formula.

Homework problems

1. If $T = 2\pi\sqrt{\frac{m}{k}}$ for a mass-spring system and we square both sides, $T^2 = \frac{4\pi^2}{k}m$. If one graphs period squared on the y-axis and mass on the x-axis, the slope of the line you got will be equal to $\frac{4\pi^2}{k}$ (and $\pi = 3.1416$, approximately). Use a slope of $1.60 \text{ s}^2/\text{kg}$ to find the k (spring constant) for this spring.
2. A mass suspended from a coiled spring is set vibrating up and down with an initial amplitude of 24.0 cm. After 20 s, the amplitude has decreased to 6.0 cm. What would be its amplitude 50 seconds after starting?
3. Respond to this comment in a few sentences: "The amplitude of a mass/spring vibration has to matter for the period. If you start pulling the mass more, it has farther to travel, so it should take longer! But we found in lab amplitude didn't make much difference. This lab makes no sense."
4. Assume there's no air resistance. You drop a dime ($m = 2.3 \text{ g}$) from a window on the 5th floor of 623 S. Wabash (that's about 16 meters high).
 - a. How much more potential energy does it have compared to a position on the ground?
 - b. If all of that energy is converted to kinetic energy as it falls, use the kinetic energy formula (since you know how much KE it has near the bottom of its fall) to find how fast the dime is going just before it hits the ground.
5. The function $y = (0.05 \text{ m}) \sin((3 \text{ s}^{-1})t)$ can describe one trial of our first lab. What is the distance between the highest and lowest points of the oscillation for this case?
6. A flatbed truck hauls a block of ice on top of which stands a penguin, as shown in the figure. Assume the block of ice is frozen solid to the bed of the truck, but the top surface of the ice is very slippery. The truck skids to a stop from an initial velocity of 20 m/s. (a) Describe what happens to the penguin using Newton's laws to justify your answer. (b) Do the same for the driver, who is wearing a seat belt.



7. A cat likes to play the following game with a chipmunk: it sits at the edge of a yard that is $w = 30$ m across. It waits as a chipmunk climbs down from a tree and moves a distance L as shown in the figure. At that point, the cat will chase the chipmunk, who turns and runs back toward the trees. If the cat's top speed is 7.5 m/s and the chipmunk's top speed is 4.5 m/s, what is the value of L for which the cat will catch the chipmunk just before it disappears into the woods?



8. A block of mass 200 grams is connected to a light, horizontal spring with a spring constant $k = 5$ N/m. It is free to oscillate on a frictionless surface, and we'll also neglect air resistance so you don't have to worry about decaying. If the block is displaced 5 cm away from equilibrium and released from rest, compute the period T of its motion. Both the homework from chapter 1 and the notes for chapter 2 give the formula for this.
9. The maximum speed of a mass-spring system is given by $A\sqrt{\frac{k}{m}}$. (a) Use this expression to compute the maximum speed of the block in the previous problem. (b) Find the maximum potential energy stored in that spring. (c) Convert that energy to kinetic energy and show using the KE formula that you get the same top speed as part (a).
10. A useful approximation skill is to know when an angle counts as "small" for situations like the pendulum problem. Here we will see what "small" means for the sine function. To do this, set your calculator to radian mode and complete the following table. We are calculating the percent difference between θ in radians and the sin of θ . The % error is

calculated by subtracting column 2 from column 1, dividing by column 2, and multiplying by 100 to get a percentage:

$$\% \text{ error} = \left(\frac{|\sin \theta - \theta|}{\sin \theta} \right) \times 100$$

θ (rad)	$\sin \theta$	% error
0.1		
0.2		
0.3		
0.4		
0.5		
0.6		
0.7		
0.8		
0.9		
1		

11. In view of the results in the above problem, how small can an angle be if we want the sine of an angle to be within 5% of the angle's value?
12. Two supposedly identical grandfather clocks are manufactured to have pendulums of length 0.993 meters. However, it is observed that one of the two clocks *loses* 26 seconds every day, i.e., it runs slow by this much. These problem steps will help us fix the problem. (a) Using physical reasoning, explain whether the pendulum of the slow clock should be shortened or lengthened? (b) Compute the period of the clock that is running on time. (c) Given that a day consists of 86,400 seconds, compute the total number of swings the on-time clock performs in a day. (d) The clock that runs slow would be perfectly on time if the day had 86426 seconds. If the machinery ensures it completes the same number of oscillations in a day as the clock that is not running slow, compute the period of the slow clock (keeping as many decimal places as needed!) (e) Now compute the true length of the slow clock's pendulum, and use your answer to determine what adjustment must be made to correct the problem.